



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

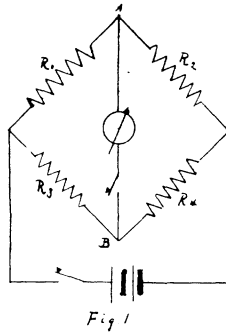
Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

MAXWELL'S METHOD OF COMPARING AN ELECTRO-STATIC CAPACITY WITH A SELF-INDUCTANCE.

By J. A. G. SHIRK, Ottawa, Kan.

WHEN direct currents are used in a Wheatstone bridge, there is a balance when the resistances of the four arms of the bridge are proportional, *i. e.*, $R_1:R_2=R_3:R_4$. (Figure 1.)

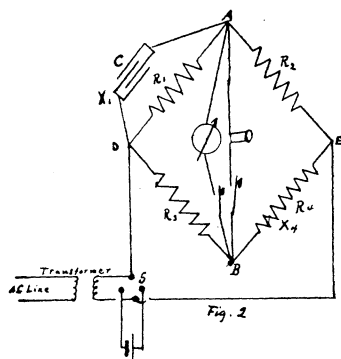


If three of these resistances are known, the fourth may be determined by adjusting the three known ones until there is a balance, and hence no deflection of the galvanometer, as the points A and B will be at the same potential.

This is the ordinary application of the Wheatstone bridge in measuring the resistance of an unknown conductor. If alternating currents are used there is a balance if the impedances of the four arms are proportional. The impedances consist of the resistances and reactances of the different branches. Let R_1 , R_2 , R_3 , and R_4 represent the resistances of the four arms, as before, and let X_1 , X_2 , X_3 , and X_4 represent the reactances of the corresponding arms. Then the impedances of the four arms become: $R_1 - jX_1$, $R_2 - jX_2$, $R_3 - jX_3$, and $R_4 - jX_4$, where $j = \sqrt{-1}$. (This method of representing the impedances of a circuit may be found in chapter V, vol. II, of the "Elements of Electrical Engineering," by Franklin and Esty.)

If three of the impedances are known, the fourth can be determined by adjusting the three known ones until there is a balance in the bridge. In order to accomplish this result arrange the bridge as in figure 2. For the direct current use a storage coil or a primary battery, the current being reduced to the desired value

by increasing the total resistance of the entire circuit. A low-voltage-pressure transformer, giving one or two volts, is best adapted for supplying the alternating current. In this case only one resistance and one reactance may be known, and therefore this method can be employed to compare two capacities, two self-inductances, or a self-inductance and a capacity. Figure 2 illustrates the method of comparing a capacity with a self-inductance.



R_1 , R_2 , and R_3 are known adjustable resistances, wound noninductively. R_4 is the resistance of the self-inductance, and is generally unknown. To find R_4 , use direct currents and balance the bridge. Then disconnect the source of direct current and balance the bridge with the alternating current, using the telephone receiver instead of the galvanometer. A balance is obtained when the sound in the receiver is a minimum. If the capacity of the condenser is given and the self-inductance is unknown, it will be easier to balance the bridge if two of the known resistances, such as R_1 and R_3 , constitute a slide-wire bridge, as their ratio can then be changed quickly and continuously and the minimum of sound can easily be determined.

If the capacity of the condenser is unknown it will be better to use a variable self-inductance at K whose resistance is constant, but if the self-inductance is constant use a slide-wire bridge for two of the resistances. If R_1 and R_3 constitute the two segments of a slide-wire bridge the condenser C should be connected in parallel with R_2 .

The following is an illustration of the use of the method in finding the capacity of a condenser C (figure 2):

Let Z_1 , Z_2 , Z_3 , and Z_4 represent the impedances of the four arms of the bridge. Since AD is a divided circuit, its impedance must

be found by a method similar to that used in finding the resistance of a divided circuit:

$$\therefore \frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{jX_1} = \frac{X_1 - jR_1}{R_1 X_1}$$

$X_1 = \frac{1}{CW}$, C being the unknown capacity of the condenser and W the angular velocity of a single rotating coil making the same number of alternations per second as the alternating currents used, and therefore equals 2π times the frequency of the current used.

From the above equation $Z_1 = \frac{R_1 X_1}{X_1 - jR_1}$. Also $Z_2 = R_2$, $Z_3 = R_3$, and $Z_4 = R_4 - jX_4$.

When the bridge is balanced for alternating currents these impedances are proportional:

$$\therefore Z_1 : Z_2 = Z_3 : Z_4 \text{ or } Z_1 Z_4 = Z_2 Z_3$$

$$\therefore \frac{R_1 X_1}{X_1 - jR_1} (R_4 - jX_4) = R_2 R_3$$

$$\therefore X_1 R_1 R_4 - jR_1 X_1 X_4 = R_2 R_3 X_1 - jR_1 R_2 R_3$$

This equation contains both real and imaginary quantities, and therefore the real quantities must be equal to each other, and the same for the imaginaries:

$$\therefore X_1 R_1 R_4 = X_1 R_2 R_3 \text{ and } R_1 X_1 X_4 = R_1 R_2 R_3$$

$$\therefore R_1 R_4 = R_2 R_3 \text{ and } X_1 X_4 = R_2 R_3$$

The first was obtained before on balancing the bridge for direct currents. From the second equation, $X_1 = \frac{R_2 R_3}{X_4}$

Since K is a self-inductance, $X_4 = L_w$

$$\therefore \frac{1}{C_w} = \frac{R_2 R_3}{L_w} \text{ and } C = \frac{L}{R_2 R_3}$$

In this experiment a variable known self-inductance was used at K , and it balanced the bridge at 20.5 mil-henrys. The values of the resistances used were $R_1 = R_3 = 400$ ohms, $R_2 = R_4 = 52.66$ ohms. The results are better when the values of the resistances are so chosen that the resistances of the two branches between D and E are nearly the same.

$$\text{For these values } C = \frac{.0205}{(52.66) 400} = .973 \text{ microfarads.}$$

The capacity of the same condenser was measured by the first method given in Carhart's "Electrical Measurements," article 107, and was found to be .979 microfarads. The results show a substantial agreement of the two methods, and considering the greater amount of time consumed and the liability of errors in this latter method, it would seem that the method of using alternating currents is to be preferred even in the measurement of capacity.